

# Machine-Learning-Based Circuit Synthesis

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**Abstract**—Multi-level logic synthesis is a problem of immense practical significance, and is a key to developing circuits that optimize a number of parameters, such as depth, energy dissipation, reliability, etc. The problem can be defined as the task of taking a collection of components from which one wants to synthesize a circuit that optimizes a particular objective function. This problem is computationally hard, and there are very few automated approaches for its solution. To solve this problem we propose an algorithm, called **Circuit-Decomposition Engine (CDE)**, that is based on learning decision trees, and uses a greedy approach for function learning. We empirically demonstrate that CDE, when given a library of different component types, can learn the function of Disjunctive Normal Form (DNF) Boolean representations and synthesize circuit structure using the input library. We compare the structure of the synthesized circuits with that of well-known circuits using a range of circuit similarity metrics.

## I. INTRODUCTION

Logic (or Boolean Function) Synthesis is a well-known problem, and is a key to developing circuits that optimize a number of parameters, such as depth, energy dissipation, reliability, etc. The problem can be defined as the task of taking a collection of components from which one wants to synthesize a circuit that optimizes a particular objective function. This problem has been addressed since Roth [1]. More recent work has focused on synthesis of circuits jointly optimizing a complex objective function [2], [3], optimization via genetic algorithms [4], [5], and on circuit re-engineering, e.g., [6].

Circuit synthesis is related to, but strictly more general than, Boolean minimization, on which there has been significant work (e.g., using the Quine-McCluskey method [7]). Rather than being given a function to optimize, we must jointly create the function and optimize it; in addition, we may want to address many other tasks in the synthesis process; such tasks include (1) optimize properties beyond just the number of gates that Boolean minimization addresses (e.g., circuit area, depth), (2) add components not present in the given function, and (3) design nested hierarchical structures in the device.

The proposed method in this paper addresses multi-level logic synthesis with machine learning techniques. While in electrical engineering, logic synthesis is motivated by design requirements, our algorithm is motivated by reverse engineering needs in addition to design and optimization goals. In reverse engineering, the logic goal function is not given, like in design problems, but should be learned by observations. In particular, the input for our problem is a

(portion of a) truth table of a given logic circuit. The output of our proposed algorithm is a model of the logic circuit consistent with all the observations.

We aim to automate the process of generating circuits from component libraries. We propose a machine learning approach. Prior work has used genetic algorithms, which do not converge well [8], [5]. We adopt a decision tree approach, and in particular, an iterative greedy algorithm that adds the most efficient component in terms of model size. Our approach is not restricted by a pre-defined library of component types but uses a library that can dynamically grow, and thus keeps the model size small.

Our approach has several important applications.

- In *reverse engineering*, engineers can shorten the process of reverse-engineering. For instance, automating this process could significantly reduce the time duration of unveiling key systems; e.g., it could emulate the reverse engineering of the ISCAS-85 benchmarks [9].
- In *model-based synthesis*, automating the process of Boolean function synthesis is needed for model-based systems. The existence of a model is a basic requirement for model-based systems. Unfortunately, in many cases such a model does not exist. We believe that automating the process will facilitate the design of model-based systems and will be able to use techniques from model-based diagnosis [10], model-based prognostics [11] and model-based problem solving [12].
- In *model-based diagnosis*, this approach can take a system function and optimize its diagnostics properties, e.g., diagnosability, fault tolerance, failure probability, etc.

Our contributions are as follows. We propose a novel machine learning approach for Boolean function decomposition for the case of multi-level logic synthesis. We propose reverse engineering of Boolean formulas rather than addressing designing problems. We cope with multiple output functions rather than a single output. We implement a method that uses a library of different component types which can be dynamically increased with new types of components. Finally, our algorithm is empirically evaluated through various of circuits.

## II. RELATED WORK

This section compares our work to prior research in a range of different areas, including Boolean optimization, function learning, and synthesis.

The task of composing a model from components to achieve a goal function is known in the electrical and computer engineering literature as logic synthesis. Logic synthesis is a process for converting a high-level specification of circuit behavior, typically register transfer level (RTL), into a *design implementation*, which can be represented in terms of logic gates.

In general, there are two kinds of logic synthesis: two-level and multi-level.

In two-level logic synthesis the goal is to represent a Boolean function by at most two gate levels between a primary input and a primary output. This can be achieved by representing the function as a DNF (in terms of the engineering literature: sum of products). Known methods for this task are Quine-McCluskey [7] to compute the exact prime implicants of the goal formula and heuristic methods like ESPRESSO [13] and MINI [14] which compute near-minimal prime implicants. A major limitation of this approach is that two-level logic circuits are of limited importance in a most real-world applications, e.g., in very-large-scale integration (VLSI) design, since most designs require multiple levels of logic.

In multi-level logic synthesis there is no restriction on the number of gates between a primary input and a primary output. Actually, most circuits in real life are multilevel. Multiple levels of gates increase the complexity of logic synthesis dramatically, so exact solutions are not practical. There are many methods to reduce a logic formula to multi-level logic. Some of the methods use only primitive gates as AND, OR and NOT, like algebraic logic optimizations and Boolean logic optimizations [15]. Others utilize decomposition of Boolean functions like BDD-based algorithms, and thus allow richer Boolean gates like XOR [16]. There are methods that utilize logic synthesis for lookup tables (LUT), which actually are not restricted by specific type of gates [17].

Many machine learning techniques propose to solve two-level logic synthesis by learning a DNF formula from observed examples. The most fundamental paper by Valiant [18] formalized the Probably Approximately Correct (PAC) model of learning from independent random examples. Bshouty et al. [19] proposed a uniform random walk model. In this model the learner's examples are not generated independently, but are produced sequentially according to a random walk on the Boolean hypercube. Muzelli et al. [20] combines the advantages of two efficient techniques, Logical Analysis of Data (LAD) and Shadow Clustering (SC). LAD enumerates in a breadth-first search all the prime implicants whose degree is not greater than a fixed maximum  $d$ . In contrast, SC is a heuristic algorithm that retrieves the most promising logical products to be included in the resulting AND-OR expression. All the above methods focus on learning DNF formulas. We, on the other hand, solve a more complicated problem of synthesizing a multi-level

logic expression. Obviously, we can reduce our approach to learn DNF formulae. As mentioned before, multi-level synthesis may be more efficient than two-level in terms of the number of gates.

Another attempt to solve the multi-level logic synthesis is by genetic algorithms. Aguirre et al. [8], [5] propose to use a multiplexer as the only design unit, defining any logic function. They first explore a feasible design and then minimize the circuit. Gan et al. [21] present the genetic-based algorithm denoted Gene Expression-based Clonal Selection Algorithm (GE-CSA), which combines the advantages of the Clonal Selection Algorithm (CSA) and Gene Expression Programming (GEP), overcoming some drawbacks of GEP. These works focus on a single output function, we on the other hand, show a machine learning approach which solves multiple logic functions in one circuit. In addition, a known drawback of genetic algorithm is the long time of convergence. Unfortunately, even the above papers demonstrate their approach only for a few simple circuits. Although Gan et al. [21] report on well convergence for two circuits, generalizing this conclusion to other circuits with more variables and more output functions is not straightforward.

Zupan et al. [3] present a new machine learning approach that infers a target function from a set of training examples. It is represented in terms of a hierarchy of intermediate, less-complex concepts and their definitions. The method is inspired by the Boolean function decomposition approach to the design of switching circuits by suboptimal heuristic algorithms. Since this algorithm is not restricted to a given set of gates, it actually tries to decompose the function to artificial sub-functions. We adopt the hierarchical approach but redesign it to solve the multi-level logic synthesis consistently by a given library of component types.

Bernasconi et al. [6] present a new type of factorization, based on a "P-circuits" approach, where a function  $f$  is projected onto three overlapping subspaces of the Boolean space  $\{0, 1\}^n$  in order to favor area minimization that avoids cube fragmentation. An approximation algorithm guarantees a constant approximation ratio of a circuit, and shows better results with respect to standard heuristic algorithms and to the classical Shannon factorization, both in terms of area and power consumption of the generated circuit.

Finally, Feldman et al. [22] present a new related problem to ours. They have implemented a General Redesign Engine (GRE), which uses model-based reasoning techniques and Boolean functional synthesis from component libraries, to automate redesign for combinational circuits. For the logic synthesis they consider fault tolerance optimization which reduces the probability of catastrophic failures. Feldman et al. do not implement a machine learning approach but a brute-force approach.

### III. CONCEPTS AND DEFINITIONS

We start by presenting a set of definitions that are designed to facilitate the exposition of algorithms for automated reasoning.

Figure 1 shows an implementation of a full-adder, represented by the function  $F(i_1, i_2, c_i) = (q \Leftrightarrow i_1 \wedge i_2) \wedge (p \Leftrightarrow i_1 \oplus i_2) \wedge (\Sigma \Leftrightarrow p \oplus c_i) \wedge (c_o \Leftrightarrow q \vee (p \wedge c_i))$ .

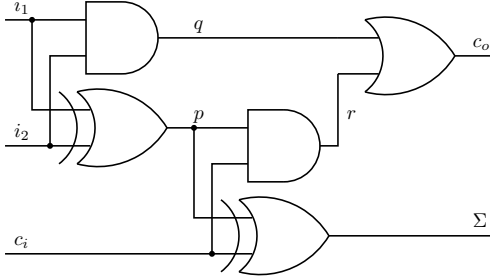


Figure 1. This full-adder is used as a running example.

**Definition 1 (Component):** A component COMP,  $\langle F, \text{IN}, \text{OUT} \rangle$ , is specified using a Boolean function  $F$  over a set of variables  $Z$ , and input/output variables,  $\text{IN}, \text{OUT} \in Z$ . Boolean functions that model components are often represented graphically, by using the same symbols as in a standard computer arithmetic schoolbook [23]. Figure 2 shows a component that implements a three-input AND gate by using two two-input ones. The Boolean function that is shown in Fig. 2 is  $F(i_1, i_2, i_3) = (o \Leftrightarrow z \wedge i_3) \wedge (z \Leftrightarrow i_1 \wedge i_2)$  where  $\text{IN} = \{i_1, i_2, i_3\}$ ,  $\text{OUT} = \{o\}$ , and  $z$  is an internal variable. We may omit specifying which variables are input and output, when that is clear from the context or from the common use (of an AND gate, for example).

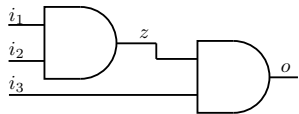


Figure 2. A component that implements a three-input AND function by using two two-input AND gates

**Definition 2 (Component Library):** A component library  $\mathcal{L}$  is defined as a set of components.

Figure 3 shows a component library consisting of a half-adder, a two-input OR gate and a two-input NAND gate. In our problem formulation, there are no restriction on the contents of the component library, i.e., it is a set of arbitrary multi-output Boolean functions. It is not necessary for a component library to contain a functionally complete subset of components (the two-input NAND gate in the component library shown in Fig. 3, for example, can be used to express any Boolean function, but that is not a requirement in our framework).

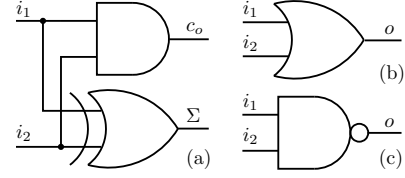


Figure 3. A component library consisting of (a) a half-adder (HA), (b) a two-input OR gate (2-OR), and (c) a two-input NAND gate (2-NAND)

**Definition 3 (System Description):** A system description SD,  $\langle \mathcal{L}, G \rangle$  is defined as a vertex-labeled and edge-labeled direct acyclic graph  $G = \langle V, E \rangle$  such that  $V = \{\text{PI} \cup \text{PO} \cup V'\}$  and if  $v \in V'$ , then  $v \in \mathcal{L}$ .

System description graphs contain a set of primary input vertices (PI), a set of primary output vertices (PO) and a vertex for each component. The graph edges are labeled with the names of the Boolean function variable names.

Figure 4 shows a system description of the full-adder circuit shown in Fig. 1, built from components drawn from the Fig. 3 library.

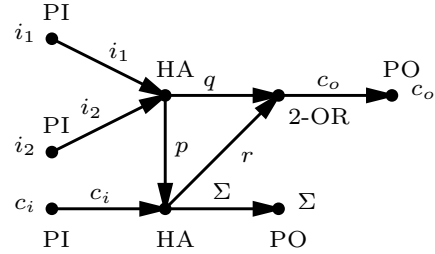


Figure 4. System description of the full-adder circuit shown in Fig. 1

A system description SD is equivalent to exactly one Boolean function as shown in the following definition.

**Definition 4 (Composition):** Given a system description SD =  $\langle \mathcal{L}, G \rangle$ ,  $G = \langle V, E \rangle$ , a composition  $C(\text{SD})$  of SD is a Boolean function  $(f_1 \circ \dots \circ f_n)(x_1, \dots, x_m)$  such that  $n = |V| - |\text{PI}| - |\text{PO}|$  and for each  $f_i \in \{f_1, \dots, f_n\}$ , there is an isomorphic function  $f'_i \in \mathcal{L}$ . The primary inputs and primary outputs of  $f_1, \dots, f_n$  are the respective edge labels in  $G$  and the internal variables in  $f_1, \dots, f_n$  are unique.

In the above definition the variables  $\{x_1, \dots, x_m\}$  are all internal variables, i.e.,  $\{x_1, \dots, x_m\} = V \setminus \{\text{PI} \cup \text{PO}\}$ .

The composition of the system description in Fig. 4, for example, is a Boolean function that is composed of two half-adders, and a two-input OR gate. The  $i_1$  and  $i_2$  inputs of the half-adder in the component library shown in Fig. 3 are renamed to  $p$  and  $c_i$  for one of the instances.

**Definition 5 (System Decomposition):** Given a component library  $\mathcal{L}$  and a Boolean function  $S$ , a system decomposition  $S^{-1}$  of  $S$  is a system description SD =  $\langle \mathcal{L}, G \rangle$  such that  $S \equiv C(\text{SD})$ .

By equivalent function we mean that, since  $S$  and  $C(\text{SD})$

have the same primary inputs and primary outputs, a valuation  $\phi(S) = 1$  iff  $\phi(C(\text{SD})) = 1$ . Note that the problem of computing if two Boolean functions are equivalent is computationally very hard.

Computing decompositions of a given Boolean function is the main problem discussed in this paper. Certain decompositions are preferable, i.e., these minimizing some optimality criterion such as number of elementary functions (number of internal nodes in the resulting system description), a cost function, etc. In this paper, the optimality criterion minimizes the number of nodes in  $G$ .

#### IV. DECOMPOSITION METRICS

We are interested in using circuit decomposition for discovering “structure” in unknown Boolean functions. To evaluate the performance of our algorithms we introduce a class of basic similarity metrics. In this case we (1) use a specified system description SD to obtain a “flat” representation, e.g., a Disjunctive Normal Form (DNF), (2) decompose the “flat” representation, obtaining a new system description SD’ and (3) use SD and SD’ to compare the metrics described next. We denote the SD graph with  $G = \langle V, E \rangle$  and the SD’ graph with  $G' = \langle V', E' \rangle$ .

We can use the graph degree distribution, where the degree of a node in a graph is the number of edges incident on that node. Since a circuit graph is directed, nodes have two different degrees, the in-degree, which is the number of incoming edges, and the out-degree, which is the number of outgoing edges. The degree distribution  $P(k)$  of a graph is the fraction of nodes in the graph with degree  $k$  (in this case we do not take into consideration the orientation of the edges). Thus if there are  $n$  nodes in total in a network and  $n_k$  of them have degree  $k$ , we have  $P(k) = n_k/n$ .

The mean degree of a graph is given by

$$\bar{P} = \frac{1}{|V|} \sum_{v \in V} |v|. \quad (1)$$

where  $V$  is the set of all nodes and  $|v|$  is the degree of a node  $v$ .

The graphs that we deal with are attributed graphs, with the nodes having a component-type attribute, which we denote as  $\lambda(v)$  for  $v \in V$ . Given this, we can define a component-type distribution, which is

$$\Lambda(V) = (\Lambda_1, \dots, \Lambda_k) = \left( \frac{|\lambda_1(v)|}{|V|}, \dots, \frac{|\lambda_k(v)|}{|V|} \right), \quad (2)$$

given component types  $1, \dots, k$  for a fixed ordering of types. For example, for the full-adder circuit with gate-type ordering (AND, OR, XOR), we have the distribution (0.4, 0.2, 0.4).

We denote the SD graph with  $G = \langle V, E \rangle$  and the SD’ graph with  $G' = \langle V', E' \rangle$ . We compare a number of graph-topology ratios, which we define as follows:

- Node Ratio:  $\equiv V/V'$ .

- Component-type distribution ratio:

$$\left( \frac{\Lambda_1(v)}{\Lambda_1(v')}, \dots, \frac{\Lambda_k(v)}{\Lambda_k(v')} \right),$$

whenever  $\Lambda_i(v)$  is not zero.

- Degree distribution ratio, where well-defined, i.e.,  $P(k) \neq 0$ .
- Average Vertex Degree Ratio  $= \frac{\bar{k}}{\bar{k}'}$ .

We use all of the above metrics to measure the quality of a decomposition. In this paper we do not supply weights for the different metrics and we do not combine them. For example, if one introduces a component cost function, it should be taken into consideration when combining the component-type distribution ratios of the different component types.

#### V. CIRCUIT DECOMPOSITION

We first discuss some general properties of the Boolean function decomposition problem and then we give an efficient algorithm for computing decompositions.

##### A. Relation to Known Decompositions

One question that arises is the type of component library that is necessary for decomposition. It turns out that we can use a library  $\mathcal{L}$  consisting of the well known *functionally complete* set of gates (Boolean operators), i.e.,  $\mathcal{L}$  can consist of the sets  $\{\text{AND}, \text{NOT}\}$ ,  $\{\text{NAND}\}$ , or  $\{\text{NOR}\}$ .

Certain decompositions are preferable, i.e., these minimizing some optimality criterion such as number of elementary functions (number of internal nodes in the resulting system description), a cost function, etc. We can thus generalize our notion of system decomposition to include a *preferred* system decomposition, which is a system decomposition that is optimal with respect to an optimality criterion  $\mathcal{O}$ .

Given these definitions, it is straightforward to reduce the problem of circuit synthesis to several well-known Boolean optimization problems. In particular:

- Consider a component library that consists of Boolean functions of the following kind:

$$f(x_1, x_2, \dots, x_n) \equiv x_1 \wedge f(\top, x_2, \dots, x_n) \vee \neg x_1 \wedge f(\perp, x_2, \dots, x_n) \quad (3)$$

for  $n = 1, 2, \dots, k$ , where  $k$  is an upper-bound for the number of variables in the functions that we want to decompose. One can show that the resulting decomposition that minimizes the number of component instances is equivalent to an optimal Shannon decomposition, i.e., the problem reduces to building a minimal-decision tree.

- If the component library consists of 2-input NAND gates only, this particular kind of function decomposition becomes equivalent to Quine-McCluskey optimization.



## B. Circuit Decomposition Algorithm

Algorithm 1 shows the main system decomposition method of this paper. The basic idea of Alg. 1 is to greedily “carve-out” component instances, starting from some subset of the primary inputs and moving toward the primary output. Alg. 1 works on single-output Boolean functions only. The input function should be given in a Disjunctive Normal Form (DNF). The core of Alg. 1 is constructing multiple decision trees, one for each component instantiation candidate added to a reduced representation of the target Boolean function. A component instantiation is selected if it minimizes the depth of the decision tree.

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### Algorithm 1: Circuit Decomposition Engine (CDE)

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**Input:**  $S$ , a Boolean function in DNF  
**Input:**  $\mathcal{L}$ , a component library  
**Result:** a system description

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1  $\langle T, \text{IN}, \text{OUT} \rangle \leftarrow \text{MAKETABLE}(S)$ ;
2 repeat
3   foreach  $\langle F, C_{\text{IN}}, C_{\text{OUT}} \rangle \in \mathcal{L}$  do
4     foreach  $X \in \text{SUBSETSOF SIZE}(\text{IN}, |C_{\text{IN}}|)$  do
5        $Z \leftarrow F(X)$ ;
6        $T' \leftarrow \text{ADDINTERNAL}(T, Z)$ ;
7        $\text{CT} \leftarrow \text{TREEINDUCER}(T')$ ;
8        $f^* \leftarrow \text{EVALUATE}(\text{CT})$ ;
9       if  $f^* < f$  then
10         $\langle f^*, Z^*, \text{CT}^* \rangle \leftarrow \langle f, Z, \text{CT} \rangle$ ;
11       $\langle T, \text{IN}, \text{OUT} \rangle \leftarrow \text{UPDATETABLE}(T, Z^*)$ ;
12 until  $\text{DEPTH}(\text{CT}^*) > 2$ ;
13 return  $\text{MAKESYSTEMDESCRIPTION}(\text{CT}^*)$ 

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$c_i$	IN		OUT	
	$i_1$	$i_2$	$c_o$	$\Sigma$
False	False	False	False	False
True	False	False	False	True
False	True	False	False	True
True	True	False	True	False
False	False	True	False	True
True	False	True	True	False
False	True	True	True	False
True	True	True	True	True

Table I  
TRUTH TABLE OF THE TARGET FUNCTION FOR THE FULL-ADDER SHOWN IN FIG. 1

Table I shows the output of MAKETABLE (line 1) for the full-adder function shown in Fig. 1. Each column in  $T$  (in the running example  $T$  is initially constructed from Table I) is an attribute and this table is a partial specification of the system description and a full representation of the target Boolean function. Each attribute (column) represents a primary input,

a primary output, or an internal variable. Note that each internal variable is also the output of a component and the name of this component can be specified in the name of the internal variable.

The main idea of Alg. 1 is to maintain a front of unused input or internal variables and to try all possible components from the component library. This front is initially constructed from all primary inputs contained in IN and later maintained in the same set of variables. Line 3 of Alg. 1 tries to use each component from the component library. Let the component chosen in line 3 has  $k = |C_{\text{IN}}|$  inputs. These  $k$  inputs are attempted to be connected to any  $k$ -subset of the variables in the set IN. These subsets are generated by the SUBSETSOF SIZE auxiliary subroutine invoked in line 4.

Consider decomposing the function of the running example whose truth table is given in Table I. CDE first draws an inverter from the component library (the order is arbitrary). It will then try to use each of the IN variables of the full-adder as an input to this inverter. Line 5 of Alg. 1 computes the values at the output of the inverter. Line 6 of Alg. 1 adds the output of the inverter to the  $T$  truth table, storing the result in the temporary  $T'$  truth table as the choice of the inverter is not final. The first  $T'$  table for our running example is shown in Table II.

$c_i$	$\neg c_i$	$i_1$	$i_2$	$c_o$	$\Sigma$
False	True	False	False	False	False
True	False	False	False	False	True
False	True	True	False	False	True
True	False	True	False	True	False
False	True	False	True	False	True
True	False	False	True	True	False
False	True	True	True	True	False
True	False	True	True	True	True

Table II  
TRUTH TABLE  $T'$  AFTER CONNECTING AN INVERTER TO THE PRIMARY INPUT  $c_i$

Each time a component is drawn from the component library and connected to unconnected input/internal variables, a decision tree is induced by the TREEINDUCER subroutine. A component is preferred if it leads to a binary decision tree with a smaller number of leaf nodes. Continuing our running example, the decision tree induces from the truth table  $T'$  shown in Table II is shown in Fig. 5.

The tree shown in Fig. 5 has eight leaf-nodes and this is the value returned by the EVALUATE function in Alg. 1. After computing the quality of the tree shown in Fig. 5, CDE, tries all other possible components. For example, after a few attempts, CDE tries connecting a XOR gate to the primary inputs  $c_i$  and  $i_1$ . The resulting truth table is shown in Table III.

Clearly, the quality of the second tree, shown in Fig. 6, and having 6 leaf-nodes is better than the first one (with 8 nodes),

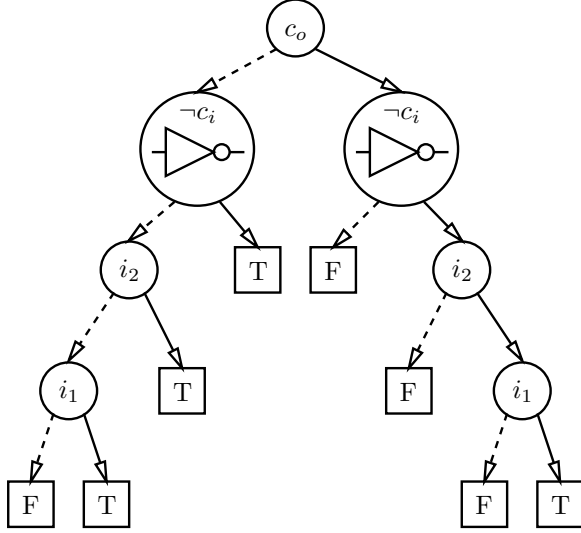


Figure 5. Binary decision diagram induced from Table II

$c_i$	$i_1$	$c_i \oplus i_1$	$i_2$	$c_o$	$\Sigma$
False	False	False	False	False	False
True	False	True	False	False	True
False	True	True	False	False	True
True	True	False	False	True	False
False	False	False	True	False	True
True	False	True	True	True	False
False	True	True	True	True	False
True	True	False	True	True	True

Table III

TRUTH TABLE  $T'$  AFTER CONNECTING AN XOR GATE TO THE PRIMARY INPUTS  $c_i$  AND  $i_1$

hence the XOR gate is preferred. The process continues until the resulting decision tree has only a root and leaf nodes, i.e., it is a stump tree. The resulting functional decomposition for our running example is shown in 7. The difference, from the original design comes from the fact that we run CDE separately for each primary output and then we combine the resulting Boolean functions. Despite that the design is very similar to the original and exhaustive checking verifies that the implemented Boolean function is equivalent to that of the original full-adder.

We next extend the results from running CDE on the full-adder to a benchmark of Boolean functions.

## VI. EXPERIMENTAL RESULTS

We have implemented CDE in Python using the Orange data mining and machine learning software suite [24] for inducing the binary decision trees. The implementation is straightforward and is a couple of hundred lines of code. We have run all our experiments on a recent Linux platform

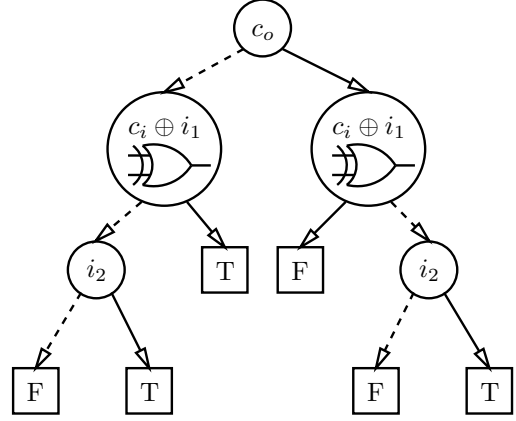


Figure 6. Binary decision diagram induced from Table III

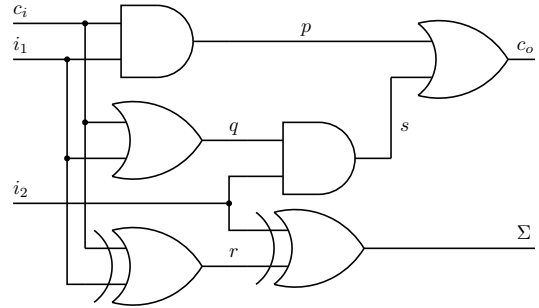


Figure 7. Decomposition of the full-adder shown in Fig. 1

based on a 2.8 GHz Intel i7 CPU and equipped with 4 GB of RAM.

### A. Benchmark

We evaluate the performance of CDE on a benchmark of combinational circuits that we introduce in this paper. The benchmark contains fifteen Boolean functions and is summarized in Table IV. The function decomposition benchmark contains small circuits and several IC designs used in the 60-s and 70-s of the twentieth century. The various functions implement common arithmetic functions such as addition and multiplication, parity checking, multiplexing and demultiplexing, etc. The 74XXX series functions have been manually decomposed by Hansen et al. [9].

The first column in Table IV is an identifier. The second column gives a brief description of the Boolean function. The remaining columns indicate if certain components (gates) are included in the Boolean functions. For example, the 2-bit adder includes an XOR gate, two- and three/input AND gates, two/input OR gates, etc.

Table V shows the size of the manually created Boolean functions from the function decomposition benchmark.

name	description	inverter	buffer	XOR	AND	NAND	OR	NOR	half-adder	1-bit adder
HA	half-adder			2	2/3					
FA1	1-bit adder			2	2/3		2		2	
FA2	2-bit adder			2	2/3		2		2	
FA4	4-bit adder			2	2		2		2	3
SUB1	1-bit subtractor	1		2	2/3		2		2	
MUX4	4-bit multiplexer	1			2/3		2/4			
DEMUX4	2-to-4 demultiplexer	1			2/3		2			
MUL2	2-bit multiplier			2	2		2		2	
MUL3	3-bit multiplier			2	2		2		2	3
PAR4	4-bit parity checker			2						
PAR6	6-bit parity checker			2						
74182	4-bit CLA	1			2/3/4		4	2/3/4		
74L85	4-bit comparator	1			2/3/4/5		5	2		
74283	4-bit adder	1		2	2/3/4/5			2/3/4/5		
74181	4-bit ALU	1	1	2	2/3/4/5	2/3/4/5	2	2/3/4		

Table IV  
CIRCUIT DECOMPOSITION BENCHMARK

### B. Experimental Results

CDE computed decompositions for 13 out of 15 benchmark instances. The algorithm could not compute decompositions for MUL3 and 74181 within the preallocated time quota of 15 min. In all successful cases the returned Boolean functions were logically equivalent to the target function.

name	V	E	PI	PO
HA	6	4	2	2
FA1	10	8	3	2
FA2	15	9	5	3
FA4	23	15	9	5
SUB1	12	10	3	2
MUX4	16	15	6	1
DEMUX4	15	11	3	4
MUL2	16	12	4	4
MUL3	32	27	6	6
PAR4	8	7	4	1
PAR6	12	11	6	1
74182	33	28	9	5
74L85	47	44	11	3
74283	50	45	9	5
74181	87	79	14	8

Table V  
CIRCUIT DECOMPOSITION BENCHMARK

CDE produces interesting results in generating functions that do not only result in all metrics equal to 1 but also being equivalent (having equivalent system descriptions). This is the case for the instances HA, SUB1, PAR4 and PAR6. The design of the 1-bit subtractor is shown in Fig. 8.

Figure 9 shows the MUX4 benchmark instance and Fig. 10 shows the DEMUX4 implementation. Fig. 11 shows a scalable n-bit adder.

The main results of CDE are summarized in Table VI. The second and third column of Table VI show the number

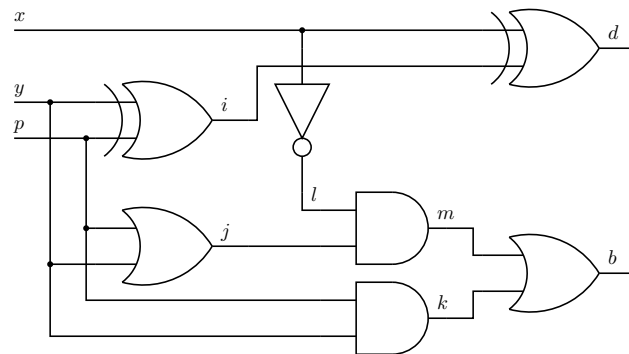


Figure 8. 2-bit subtractor

of nodes and edges, respectively, of the system description returned by Alg. 1. The ratio of these sizes to the original graph sizes shown in Table V are given in the forth and fifth columns of Table VI. We can see that these values are often close to 1 which means that the graphs are of similar size. The rightmost column of Table VI shows the time in seconds it takes for CDE to decompose the target Boolean function.

Table VII shows the distribution of the components and the target and synthesized system descriptions. In general there are many complete functional sets and the choice of CDE is driven by, e.g., the ordering in the component library when breaking ties due to equivalent quality of the Boolean decision tree. Because of this potential equivalence of component library and gates, CDE may replace, for example, NAND components with OR components and inverters. The results in Table VII show that the performance of CDE decreases with increasing the size of the target Boolean function. The ratios shown in this table are 1 if there is a 1:1 equivalence in gate numbers between the original

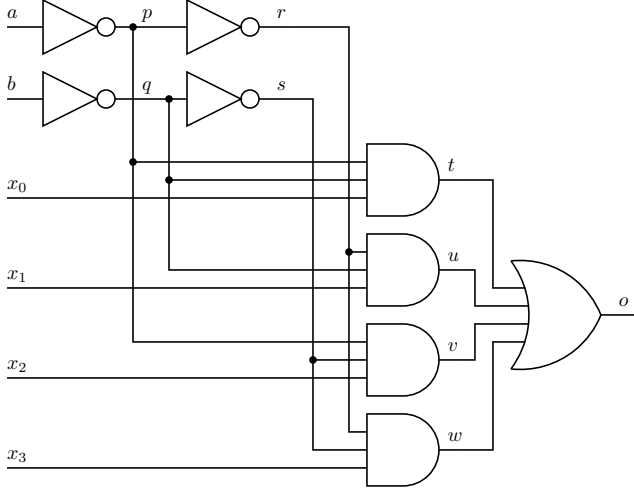


Figure 9. 4-bit multiplexer

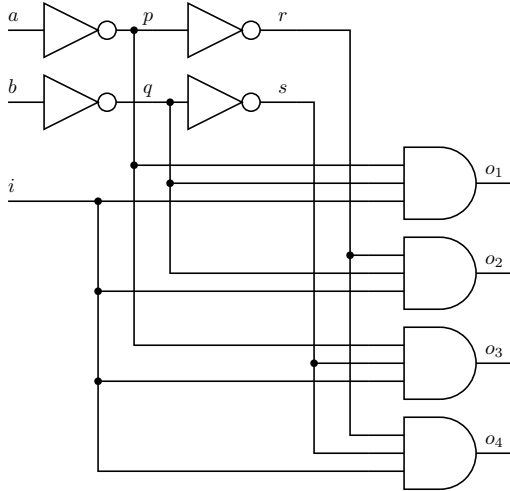


Figure 10. 2-to-4 demultiplexer

circuit and the synthesized circuit; ratios less than 1 indicate that the synthesized circuit has more of that gate type than the original circuit.

## VII. CONCLUSIONS

In this paper we have formulated the problem of Boolean logic synthesis (or circuit decomposition) from generic component libraries. This problem is computationally very hard and, depending on the functional completeness of the component library, may have no solution. We have designed and implemented the CDE algorithm that is based on machine learning, i.e., it greedily “carves-out” component instances from the target function (i.e., the function to be synthesized) until some termination criterion is met. Our design is based on the idea to use a generic version of the Shannon decomposition (which is related to building

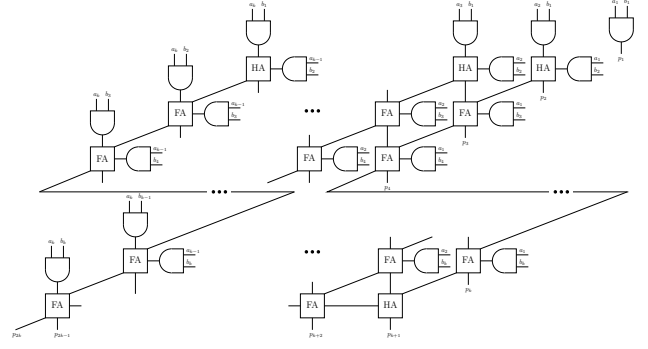


Figure 11.  $n$ -bit multiplier

name	$ V' $	$ E' $	$ V / V' $	$ E / E' $	time [s]
HA	6	4	1	1	0.59
FA1	11	9	0.91	0	0.64
FA2	23	20	0.65	0	3.17
FA4	49	36	0.47	0	119.09
SUB1	12	10	1	1	0.66
MUX4	19	18	0.84	0	11.91
DEMUX4	17	13	0.88	0	1.23
MUL2	19	15	0.84	0	1.32
MUL3	-	-	-	-	-
PAR4	8	7	1	1	0.35
PAR6	12	11	1	1	0.92
74182	52	47	0.63	0	36.36
74L85	90	87	0.52	0	532.20
74283	108	103	0.46	0	135.17
74181	-	-	-	-	-

Table VI  
DECOMPOSED BOOLEAN FUNCTIONS

decision trees) as a heuristic in solving the more difficult generalization of decomposing functions in terms of *arbitrary* component (function) libraries. The last feature sets apart our work from Boolean function minimization such as minimal covers, optimal decision diagrams, etc.

To verify the validity of our method we propose a benchmark of combinational circuits. In addition, we have identified a set of basic graph similarity metrics which we use for validating our algorithm. In some cases, CDE reverses a Boolean function that has been constructed manually. In the rest of the cases CDE computes function decompositions that have number and types of component similar to the original, manually created Boolean functions. Our approach is clearly many orders of magnitude faster than the trivial brute-force approach that terminates only for Boolean functions of a very few variables.

This work is introductory in a sense that, to the best of our knowledge, there is no in-depth *algorithmic* analysis of the problem of logic synthesis. As a future work we plan (1) to improve the CDE algorithm, (2) to formulate more problems related to logic synthesis, (3) to identify and



name	inverters	XOR	AND	OR
HA	-	1	1	-
FA1	-	1	1	0.5
FA2	-	1	0.67	0.4
FA4	-	0.22	0.27	0.24
SUB1	1	1	1	1
MUX4	2	0	0.8	0.33
DEMUX4	1.33	-	1	0
MUL2	0	2	1	0
MUL3	-	-	-	-
PAR4	-	1	-	-
PAR6	-	1	-	-
74182	0.06	-	1.18	0.18
74L85	0.29	0	1.27	0.08
74283	0.55	0.29	0.4	0
74181	-	-	-	-

Table VII  
COMPONENT DISTRIBUTION METRIC

implement more metrics for evaluating the performance of algorithms. To improve the CDE algorithm we intend to implement guided back-jumping, exploration of hierarchy and active learning of decomposed sub-functions. Problems related to the problem of circuit synthesis is counting the number of decompositions and multi-parameter optimization of decompositions. Finally, metrics that can improve our evaluation include identification of maximal isomorphic subgraphs and similar.

Given the simplicity of our approach, it shows great promise given that there are many optimizations that can be introduced. Such optimizations include introducing better objective functions, applying heuristics to the simple greedy method, and learning sub-function component models that can be quickly substituted during the decomposition process.

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